Mathematical Model of Cornea Shape after Descemets Stripping Endothelial Keratoplasty

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ABSTRACT

In this paper, a mathematical model is constructed to investigate the change in eyes with respect to time after Descemet's Stripping Endothelial Keratoplasty. Three major refractive parts of the anterior eye are under discussion which mostly gets affected during surgery, i.e., axial length, central corneal thickness and corneal curvature. Also, the refractive power of the eye will be calculated. Two new formulas have been derived to describe the corneal curvature and refractive power of an eye respectively. The stability of this model is proven and validity is shown by comparison of numerical simulation and data available in different research papers.

Keywords: Axial Length, Corneal Thickness, Corneal Curvature, Differential Equations, Refractive Power, Stability Analysis.

INTRODUCTION

Mathematical biology has gained lot of interest of researchers in different fields in a very limited time. Mathematical biology represents two sorts of things mainly:

- To convert a biological phenomenon in equation form and deal with such complex situations that occurs in real life.
- To develop a new mathematical technique to understand the mathematical models and their behaviours. In short, mathematical biology deals with both the products and the processes of mathematical modelling. Many mathematical models have yet been formulated for different biological, ecological, botanical, zoological, genetics phenomenon etc. In human body, eye is the smaller, sensitive part and therefore its modelling is sensitive as well. A lot of research work has been done on modelling the structure of the eye, its diseases, and its treatments. As in (Gonzalez & Fitt, 2003) the theoretical mechanics of eye and its function is discussed, in detail. In (Mahathanthila, 2017), a numerical model is formed for the drug delivery process in human eye by considering the anterior components of the eye and suggested that drug is consumed far better when the ocular temperature and environmental temperature are almost equal. The optical system of the eye has also been analysed by developing a mathematical model using ocular wavefront tomography (Wei & Thibos, 2008). Human eye is mainly made up of two chambers anterior and posterior chamber and therefore, both chambers have different types of diseases and treatments. Since cornea belongs to the anterior chamber therefore its diseases and treatments have local impact just in the anterior chamber only, but since anterior chamber plays a vital role in eye refraction therefore due to the damage in cornea the whole eye vision gets disturbed and need an urgent care or treatment. Scarring of the cornea due to the eye infection, keratoconus, eye herpes, fungal keratitis, and growth of eyelashes inward, complications of eye surgery or other conditions are treated with Endothelial Keratoplasty such as DSEK, DSEAK, DMEK or PRK, LASIK etc. The first scientist that conducted engineering research into the cornea was H. von Helmholtz during the 19th century (von Helmholtz & Southall, 1962). A successful refractive surgery with a fitting contact lens not only depends on the models that describe

but also on corneal topography (Mahavier & Hunt, 2006). For such models, it is very important to describe the shape of cornea very precisely. Several mathematical models of cornea have already been discussed in literature. Some of them are based on revolution surfaces such as parabolas and ellipses. Few researchers believe that generalization of these conics with varying eccentricity can describe cornea better (Rosales *et al*., 2009). It is also worth mentioning, apart from purely theoretical mathematical eye models there are also physical replicas of average human eyes (Bakaraju *et al*., 2010). In (Plociniczak *et al*., 2014) a non-linear, two-point boundary value ordinary differential equation was formed to describe the corneal curvature under a static force balance and its numerical solution was obtained by transforming it into a parabolic partial differential equation by using a method of lines. Authors of (Plociniczak, *et al*., 2014) used another technique known as meshless method based radial basis function for analysing the solution of corneal curvature model. (Khalid & Fareeha, 2020) developed a mathematical model named DSEK model by using system of ordinary differential equations to show the procedure of refractive surgery as a dynamical system. Millions of research papers and theories being published since decades, describing parts of eye, its disease and treatments but all mathematical models or procedures to study eye, its movement and diseases are based on complex geometrical concepts which are not simple enough to understand for a practicing ophthalmologist without prior knowledge of mathematical modelling, basic concepts of trigonometry and geometry see (Rosales *et al*., 2009) (Gonzalez & Fitt, 2003).

OBJECTIVE

To develop a simple mathematical model to analyse the change in the behaviour of anterior part of the eye after DSEK.

METHODOLOGY

To understand the mathematical formulation of any dynamical system in the simplest possible way is through a flow chart. Figure **1** describes the workflow of the anterior part of an eye when a light ray enters from air medium to the cornea, the anterior chamber depth and vice versa.

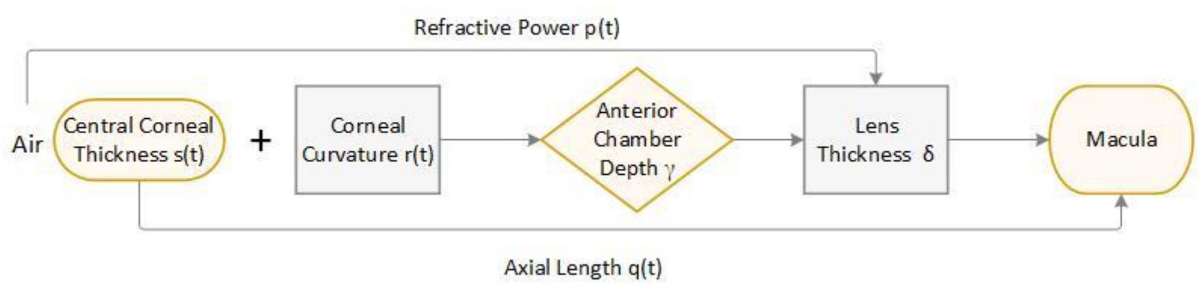

Figure 1. A well explained simple flowchart to show the refraction process in the anterior part of human eye.

Based on this information a new mathematical model has been developed using the concepts of ordinary differential equation that can accurately predict the outcome of DSEK. Since a mathematical model needs to be verified mathematically, hence its stability analysis have been discussed. Then the numerical analysis has also been presented that shows the accuracy of this model when compared with the research papers published by ophthalmologists.

RESULT

This model can predict the outcome of a refractive surgery very accurately, if provided with more accurate initial data. Also, two new formulas have been developed to examine the corneal curvature after refractive surgery and to calculate the refractive power of eye.

CONCLUSION

In this work, a new mathematical model is developed to study the changes in behaviour of anterior parts of eye. It is observed in this study that central corneal thickness, corneal curvature, and axial length are the parts that get mostly affected due to refractive surgery. Also, it is observed that the refraction of eye gets stable in almost 72 hours. It is the time when a patient needs more care, the graft heals up and swelling worn off. Mathematically, for the first time a mathematical model is developed that studies this combination of eye parts and gives accurate results when compared to ophthalmological research papers. Its stability is also discussed in this work. The stable and unstable regions have also been showed above. All the eigen values obtained should either be negative or real for a stable region and for this model, it will only be stable if $r > 0$ otherwise it will be unstable or saddle point. This model can predict the outcome of a refractive surgery very accurately, if provided with more accurate initial data. Also two new formulas have been developed to examine the corneal curvature after refractive surgery and to calculate the refractive power of eye.

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