Projective Mapping of Grassmannian Complex to Tangent Complex in Weight 2 and Weight 3

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ABSTRACT

We consider projective differential mapping in the Grassmannian complex and take the tangent to the Bloch-Suslin complex for weight q = 2 and tangent to the Goncharov complex for weight q = 3. We define morphisms from the Grassmannian complex to the tangent of the Bloch-Suslin and Goncharov complexes. We show that the introduced morphisms are well-defined and that the diagrams (by connecting two complexes) are commutative. We use the functional equations of the groups $TB_2(L)$ and $TB_3(L)$ as our main tool.

Keywords: Projective Mapping, Tangent Complex, Cross Ratio, Grassmannain Complex.

INTRODUCTION

Cathelineau describes the tangential version of the polylogarithm. He defines the tangent group $TB_2(F)$ and also writes its functional equations. Siddiqui defines the v^{th} truncated polynomial ring by $F[\epsilon]_{\mathcal{V}}$:=$F[\epsilon]/\epsilon^{v}$ over the n-dimensional affine space $A''_{F[\epsilon]\mathcal{V}n}$. He also finds the Siegel cross-ratio, its identities, and the triple ratio over the n-dimensional affine space. He describes the projected five-term relation in $TB_2(F)$ and defines the geometric configurations for the tangent complex to the Bloch-Suslin complex.

He finds the morphisms between the tangent and Grassmannian complexes and introduces the $TB_3(F)$ group, which satisfies the three-term relation, inversion relation, and also reproduces Cathelineau's 22-term relation. He defines commutativity between the Grassmannian complex and the tangential complex of weights 2 and 3, and also proves that the Grassmannian complex forms a bicomplex with Cathelineau's tangential complex. He also defines $TB_n(F)$, which is analogous to $B_n(F)$for any n.

In this work, we aim to find the projective morphisms between the Grassmannian complex and the tangent complex in weights 2 and 3 and show that the result is commutative.

OBJECTIVE

The main objectives of this research are:

- To identify the projective morphisms between Grassmannian complexes and tangent complexes of weights 2 and 3.
- To verify the commutativity between the Grassmannian complex and the tangent complex for weights 2 and 3.

METHODOLOGY

First we will define projective maps between Grassmannian and tangential complex in weight 2 and 3 and then we will verify their well definiteness and commutativity.

For weight 2

$$\begin{tikzcd} C_4(\mathbb{A}^2_{F[\varepsilon]_2}) \arrow[r, "d'"] \arrow[d, "\pi^2_{1,\varepsilon}"] & C_3(\mathbb{A}^1_{F[\varepsilon]_2}) \arrow[d, "\pi^2_{0,\varepsilon}"] \\ T\mathcal{B}_2(F) \arrow[r, "\partial_\varepsilon"] & F \otimes F^\times \oplus \wedge^2 F \end{tikzcd} \qquad \text{a}$$

where d' , $\pi^2_{1,\varepsilon}$ and ∂_ε are known functions. So, our first work is to define the map $\pi^2_{0,\varepsilon}$ then will prove that the above diagram is commutative by using siegal cross ratio,two term and inversion relation.

For weight 3

$$\begin{tikzcd} C_6(\mathbb{A}^3_{F[\varepsilon]_2}) \arrow[r, "d'"] \arrow[d, "\pi^3_{2,\varepsilon}"] & C_5(\mathbb{A}^2_{F[\varepsilon]_2}) \arrow[r, "d'"] \arrow[d, "\pi^3_{1,\varepsilon}"] & C_4(\mathbb{A}^1_{F[\varepsilon]_2}) \arrow[d, "\pi^3_{0,\varepsilon}"] \\ T\mathcal{B}_3(F) \arrow[r, "\partial_\varepsilon"] & (T\mathcal{B}_2(F) \otimes F^\times) \oplus (F \otimes \mathcal{B}_2(F)) \arrow[r, "\partial_\varepsilon"] & (F \otimes \wedge^2 F^\times) \oplus (\wedge^3 F) \end{tikzcd} \qquad \text{b}$$

In above diagram the known functions are ∂_ε , $\pi^3_{2,\varepsilon}$ and d' . So we will define the maps $\pi^3_{0,\varepsilon}$ and $\pi^3_{1,\varepsilon}$ then show that left and right squares of the above diagram are commutative.

RESULT AND CONCLUSION

For weight 2

$$\pi^2_{0,\varepsilon} \circ d' = \partial_\varepsilon \circ \pi^2_{1,\varepsilon} \qquad \text{A}$$

For weight 3

$$\pi^3_{0,\varepsilon} \circ d' = \partial_\varepsilon \circ \pi^3_{1,\varepsilon}$$
$$\pi^3_{2,\varepsilon} \circ \partial_\varepsilon = d' \circ \pi^3_{1,\varepsilon} \qquad \text{B}$$

In this study we connect Grassmannain complex and tangential complex projectively and find vertical and horizantal morphisms, the above relation A for weight 2 shows that the projective mapping between Grassmannian complex to tangent complex in weight 2 is commutative and diagram [a] is projectively commutative. Also relation B for weight 3 verfiy that the product of projective maps of right and left square of the above diagram [b] is commutative.

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