Alphabet Chaos: Discovery of a Chaotic System with D–M–A Phase Portraits and its Application to Image Encryption

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ABSTRACT

This paper introduces a newly discovered chaotic system that exhibits unique phase portraits resembling the English alphabets D, M, and A. The system's nonlinear dynamics are thoroughly investigated through a series of mathematical analyses, including Lyapunov Exponents (LE), equilibrium point and eigenvalue analysis, bifurcation diagrams, and Poincaré sections, confirming its rich chaotic behavior and sensitivity to initial conditions. Furthermore, the 0–1 chaos test is employed to validate the chaotic nature of the model. To demonstrate its practical significance, the proposed chaotic system is applied to the field of image encryption, exploiting its high randomness and unpredictability for secure data transmission. The results highlight the system's potential as a robust foundation for cryptographic and secure communication applications.

Keywords: Chaos, Nonlinear Dynamics, Encryption, Random Numbers.

INTRODUCTION

Chaotic systems have long been a cornerstone of nonlinear dynamics, offering profound insights into complex behaviors in fields ranging from physics and engineering to cryptography. The discovery of new chaotic systems expands this domain by revealing novel attractors and phase portraits that exhibit unique geometric patterns, such as shapes resembling English alphabets. This study introduces a novel chaotic system with phase portraits mimicking the letters D, M, and A, providing fresh context for understanding sensitivity to initial conditions and rich dynamical properties. Such systems hold significant potential in practical applications, particularly image encryption, where high randomness ensures robust security against unauthorized access.

OBJECTIVES

The primary goals of this study are to introduce and rigorously analyze a newly discovered chaotic system with distinctive alphabet-like phase portraits (D, M, and A). Specifically, it aims to confirm the system's chaotic nature through comprehensive mathematical tools, explore its nonlinear dynamics, and demonstrate its applicability in image encryption to leverage unpredictability for secure data transmission. The purpose is to establish this system as a reliable foundation for cryptographic protocols and secure communication technologies.

METHODOLOGY

The research employs a multifaceted analytical approach to investigate the chaotic system. Equilibrium points are identified and analyzed via eigenvalue calculations to assess stability. Lyapunov Exponents (LE) are computed to quantify chaos and sensitivity to initial conditions. Bifurcation diagrams illustrate parameter-dependent transitions in system behavior, while Poincaré sections provide cross-sectional views of the attractors. The 0–1 chaos test is applied as a binary validation tool for chaotic versus regular dynamics. For practical demonstration, the system's chaotic sequences are integrated into an image encryption algorithm, utilizing pseudorandom number generation derived from the trajectories to scramble pixel values and ensure encryption efficacy.

KEY FINDINGS

The system exhibits confirmed chaotic behavior, evidenced by positive Lyapunov Exponents indicating exponential divergence of trajectories and extreme initial condition sensitivity. Equilibrium point analysis reveals unstable fixed points, supporting multistability. Bifurcation diagrams display period-doubling routes to chaos, and Poincaré sections confirm dense, non-repeating structures in the attractors. The 0–1 test yields a value near 1, unequivocally validating chaos. In image encryption applications, the system achieves high entropy and randomness, resulting in encrypted images resistant to statistical attacks, with successful decryption only via correct keys.

CONCLUSION

This study successfully introduces and validates a novel chaotic system with unique D, M, and A-shaped phase portraits, demonstrating its rich dynamics through LE, eigenvalues, bifurcations, Poincaré sections, and the 0–1 test. The findings underscore its superiority in generating unpredictable sequences, making it highly effective for image encryption and secure communications. Implications include enhanced cryptographic tools for data protection in digital eras. Future recommendations involve hardware implementation for real- time applications and exploration of synchronization in multi-system networks.

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